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The effect of heat conduction resistances of tubes and shells on transient behaviour of heat exchangers

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Abstract—With respect to radial heat conduction in tube and shell walls as well as heat loss from the shell to the environment, a predicting method for transient behaviour of counter and parallel heat exchangers is developed. Some parameters such as the Biot number (Bi) and the Fourier number (Fo) of estimating the effect of radial heat conduction are proposed. If $Bi > 0.05$, radial heat conduction will remarkably affect dynamic characteristic of heat exchangers and it should be taken into account. In addition, an approximate approach of considering radial heat conduction is presented, which reduces much of computing time. There exists satisfactory coincidence between accurate and approximate treatment of radial heat conduction.

INTRODUCTION

BESIDES stationary design and rating procedures, dynamic modelling of thermal performance of heat exchangers is incurring more and more attention because of the increasing demand for real-time control and optimal operation of the whole plant. One can find plenty of publications on transient behaviour of shell and tube heat exchangers [1–4]. In these papers, there exists a common feature that the effect of heat conduction resistance of tubes and shells on transient response to inlet disturbances has been neglected. The problem of how great the effect of wall heat conduction resistance to transient behaviour of heat exchangers is still unclear. To the best of our knowledge, so far few have quantitatively estimated this influence in detail. Regardless of the shell, Ma *et al.* [5] considered the effect of heat conduction resistance of a tube in their work with the plug-flow model, but the description is indistinct and only the final result in the frequency-domain is listed. It can be inferred from their result that they might approximately treat a tube wall as an infinite wall, which should be allowed only for high frequency oscillations or short time intervals since the curvature radius of a tube is generally not big enough to consider a tube wall as an infinite wall. In addition to the tube wall, heat capacity and heat conduction resistance as well as heat transfer between the shell and the environment exert influence on transient behaviour of exchangers, which have been ignored in most publications.

A tube wall and shell will damp transient response of exchangers to any entrance disturbances and such damping effects are dependent on ratios of their heat capacities to those of fluids. Heat exchange between hot and cold fluids and heat loss to the environment are controlled by the convective heat transfer resistance R_{conv} between the fluid and the wall and heat conduction resistance R_{cond} of the wall. Therefore, a suitable quantity to describe the effect of heat conduction of a wall is the Biot number Bi which is defined as the ratio of R_{cond} to R_{conv}. A larger value of Bi means a greater effect of heat conduction resistance. The usual treatment method of neglecting R_{cond} corresponds to the ideal case $Bi = 0$.

Based on the dispersion model, this article attempts to analyse the effect of heat conduction in a tube wall and in a shell on dynamic process of counter or parallel heat exchangers. Heat capacities of both fluids and solid components are included. Heat loss from the shell to the environment is involved. Any entrance variation of temperatures of both fluids is allowed. For the sake of reduction of computational time, the Laplace transform and numerical inversion techniques are used.

GOVERNING EQUATIONS OF TRANSIENT PROCESS

Heat conduction in the tube walls and the shell of exchangers embodies both transverse and longitudinal direction. In general, longitudinal heat conduction in the solid wall is negligible [6]. Therefore, here attention is focused on transverse heat conduction. A counter or parallel heat exchanger is sche-

NOMENCLATURE

a_w, a_s thermal diffusivity of tube wall, shell $[\mathrm{m^2\,s^{-1}}]$
a ratio of inside and outside radii of a tube wall, r_2/r_1
A heat transfer area $[\mathrm{m^2}]$
b ratio of inside and outside radii of the shell, r_{si}/r_{so}
Bi Biot number
C heat capacity $[\mathrm{J\,K^{-1}}]$
$f_1(z), f_2(z)$ any inlet temperature changes
$F_1(s), F_2(s)$ transformed forms of $f_1(z)$ and $f_2(z)$ in the Laplace image domain
Fo Fourier number
h heat transfer coefficient $[\mathrm{W\,m^{-2}\,K^{-1}}]$
L length of a heat exchanger [m]
l distance from the entrance of shellside fluid [m]
Pe Péclet number
r radius of a tube or the shell [m]
R ratio of thermal flow rates of both fluids
R_{c1} $C_1/C_2 = 1/R_{c2}$
R_s C_s/C_w
R_w $C_w/(C_1+C_2)$
s variable of Laplace transform
t dimensionless temperature, $(\theta-\theta_0)/(\theta_r-\theta_0)$
T transformed form of t in the Laplace image domain
$\dot{W}$ thermal flow rate $[\mathrm{W\,K^{-1}}]$
x dimensionless distance, l/L
z dimensionless time, τ/τ_{r1}.

Greek symbols
δ thickness of a tube wall or of the shell [m]
η dimensionless space-variable in the shell, r/r_{so}
θ temperature [K]
θ_0, θ_r initial and reference temperatures [K]
λ thermal conductivity $[\mathrm{W\,m^{-1}\,K^{-1}}]$
ξ dimensionless space-variable in a tube wall, r/r_1
τ time [s]
τ_r residence time of a fluid in the heat exchanger [s].

Subscripts
1, 2 shell and tubeside fluids
i inside
o outside
s shell
w tube wall.

matically shown in Fig. 1. Based on the assumption of constant properties, the dimensionless governing equations for the shellside and tubeside fluids are respectively derived as follows:

$$\frac{1}{Pe}\frac{\partial^2 t_1}{\partial x^2} - \frac{\partial t_1}{\partial x} - \frac{\partial t_1}{\partial z} - U_1(t_1 - t_{w1}) - U_s(t_1 - t_{si}) = 0 \tag{1a}$$

$$\left.\begin{aligned} t_1 - \frac{1}{Pe}\frac{\partial t_1}{\partial x} &= f_1(z) \quad \text{at } x = 0 \\ \frac{\partial t_1}{\partial x} &= 0 \quad \text{at } x = 1 \end{aligned}\right\} \tag{1b}$$

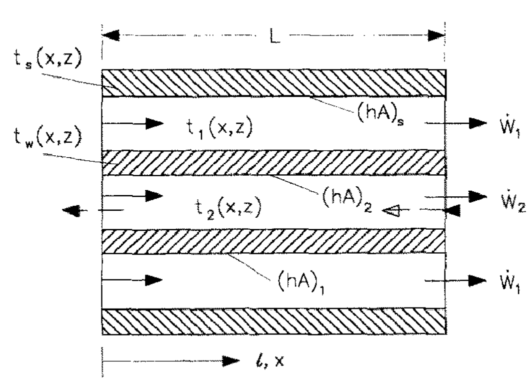

FIG. 1. Scheme of a counter or parallel heat exchanger.

$$t_1(x,z) = g_1(x) \quad \text{at } z = 0 \tag{1c}$$

$$\pm\frac{\partial t_2}{\partial x} + R_\tau \frac{\partial t_2}{\partial z} + U_2(t_2 - t_{w2}) = 0 \tag{2a}$$

$$\left.\begin{aligned} t_2 &= f_2(z) \quad \text{at } x = 0 \text{ for parallel flow} \\ t_2 &= f_2(z) \quad \text{at } x = 1 \text{ for counter flow} \end{aligned}\right\} \tag{2b}$$

$$t_2(x,z) = g_2(x) \quad \text{at } z = 0 \tag{2c}$$

where $U_1 = (hA)_1/\dot{W}_1$, $U_2 = (hA)_2/\dot{W}_2$, and $U_s = (hA)_s/\dot{W}_1$. According to the definition of the number of transfer units [7], U_1 and U_2 can be regarded as the shell and tubeside number of transfer units, respectively. Equation (1) is derived from the dispersion model and the first term on the left side describes the effect of shellside flow maldistribution. t_{w1} in equations (1) and t_{w2} in equations (2) are dimensionless wall temperatures on the sides of fluid 1 and fluid 2, respectively. In view of the lateral heat conduction in a tube wall, the following system of partial differential equations is derived:

$$\frac{\partial^2 t_w}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial t_w}{\partial \xi} = \frac{\alpha^2}{Fo_w}\frac{\partial t_w}{\partial z} \quad a < \xi < 1 \tag{3a}$$

$$-\frac{\partial t_w}{\partial \xi} + \alpha\, Bi_2(t_w - t_2) = 0 \quad \text{at } \xi = a \tag{3b}$$

$$\frac{\partial t_w}{\partial \xi} + \alpha Bi_1(t_w - t_1) = 0 \quad \text{at } \xi = 1 \tag{3c}$$

$$t_w(\xi, z) = g_w(\xi) \quad \text{at } z = 0 \tag{3d}$$

where $\alpha = r_1/\delta_w$ and $Bi_1 = h_1 \delta_w/\lambda_w$ or $Bi_2 = h_2 \delta_w/\lambda_w$ is the Biot number which is composed of the convective resistance on the side of fluid 1 or fluid 2 and the conduction resistance of a tube wall. Obviously, the case of $Bi_1 \to 0$ and $Bi_2 \to 0$ means that no temperature gradient in a wall exists and $t_{w1} = t_{w2} = t_w$, which is one of the most common assumptions for dynamic analysis. In fact, Bi_1 and Bi_2 assume finite values other than 0 (especially for turbulent flow and phase-change flow) and the assumption of zero conduction resistance is violated. The Fourier number $Fo_w = a_w \tau_{r1}/\delta_w^2$ prescribes the temperature response of a tube wall to any temperature changes of fluids. Therefore, both $Bi_{1,2}$ and Fo_w characterize transient heat transfer process of heat exchangers. Obviously, there exist the following relationships:

$$Bi_2 = Bi_1 \frac{U_2}{U_1} R_2 \left(1 + \frac{1}{\alpha a}\right)$$

and

$$Fo_w Bi_1 = \frac{1+a}{2} \frac{U_1}{R_w} \frac{1}{1+R_{c2}}. \tag{4}$$

The inner wall temperature t_{si} of the shell in equation (1a) is given by

$$\frac{\partial^2 t_s}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial t_s}{\partial \eta} = \frac{\beta^2}{Fo_s}\frac{\partial t_s}{\partial z} \quad b < \eta < 1 \tag{5a}$$

$$-\frac{\partial t_2}{\partial \eta} + \beta Bi_{si}(t_s - t_1) = 0 \quad \text{at } \eta = b \tag{5b}$$

$$\frac{\partial t_s}{\partial \eta} + \beta Bi_{so} t_s = 0 \quad \text{at } \eta = 1 \tag{5c}$$

$$t_s(\eta, z) = g_s(\eta) \quad \text{at } z = 0 \tag{5d}$$

where $\beta = r_{so}/\delta_s$, $Bi_{si} = h_{si}\delta_s/\lambda_s$, $Bi_{so} = h_{so}\delta_s/\lambda_s$ and $Fo_s = a_s \tau_{r1}/\delta_s^2$. Boundary condition (5c) describes heat loss of the shell to the environment and the heat loss approaches zero if $Bi_{so} \to \infty$. Similarly,

$$Fo_s Bi_{si} = \frac{1+1/b}{2(1+R_{c2})} \frac{U_s}{R_s R_w}. \tag{6}$$

As for other solid components such as segments and end plates, their heat capacities can be approximately incorporated in the heat capacity of tubes to take their effects into account. The combination of differential equations (1)–(3) and (5) prescribes dynamic processes taking place in counter or parallel heat exchangers. These equations can be separately solved by means of the Laplace transform in the transform-domain, although they are closely conjugated through the energy balance among fluid streams and solid components.

SOLUTIONS IN THE LAPLACE TRANSFORM-DOMAIN

Applying the Laplace transform to equation (3a), one can find its general solution

$$T_w = c_1 I_0(\sigma_w \eta) + c_2 K_0(\sigma_w \eta) \tag{7}$$

where I_0 and K_0 are modified Bessel functions of zero order and $\sigma_w = (s\alpha^2/Fo_w)^{1/2}$. Solution (7) is valid for uniform initial condition $g_w(\eta) = 0$. If $g_w(\eta) \neq 0$, one can use the parameter variation method to find a general form of solution to equation (3a). According to the given boundary conditions (3b) and (3c), the transformed temperatures T_{w1} and T_{w2} of t_{w1} and t_{w2} are determined as

$$T_{w1} = \alpha_1 T_1 + \alpha_2 T_2 \quad \text{and} \quad T_{w2} = \beta_1 T_1 + \beta_2 T_2 \tag{8}$$

where α_1, α_2, β_1 and β_2 are functions of zero- and first-order modified Bessel functions and the Biot number. Their forms are confined by conditions (3b) and (3c). Similarly, application of the Laplace transform to equations (5a) yields

$$T_s = d_1 I_0(\sigma_s \xi) + d_2 K_0(\sigma_s \xi) \tag{9}$$

where $\sigma_s = (s\beta^2/Fo_s)$. The transformed temperature T_{s1} of t_{s1} is obtained as

$$T_{s1} = \gamma T_1 \tag{10}$$

where γ is determined from boundary conditions (5b) and (5c). By inserting equations (8) and (10) into the transformed forms of equations (1) and (2), one can find the transformed temperatures T_1 and T_2 of both fluids in the Laplace transform-domain. Therefore, equations (1) and (2) are transformed as follows, in view of the uniform initial conditions:

$$\frac{d^2 T_1}{dx^2} - Pe\frac{dT_1}{dx} - a_{11}T_1 - a_{12}T_2 = 0 \tag{11}$$

$$\frac{dT_2}{dx} - a_{21}T_1 - a_{22}T_2 = 0 \tag{12}$$

where

$$a_{11} = Pe(s + U_1 + U_s - U_1\alpha_1 - U_s\gamma), \quad a_{12} = -PeU_1\alpha_2$$

$$a_{21} = \pm U_2\beta_1, \quad \text{and} \quad a_{22} = \pm(U_2\beta_2 - R_\tau s - U_2).$$

The transformed temperatures T_1 and T_2 are determined subject to the related boundary conditions:

$$T_1(x,s) = A_1 \exp(\lambda_1 x) + A_2 \exp(\lambda_2 x) + A_3 \exp(\lambda_3 x) \tag{13}$$

$$T_2(x,s) = A_2 e_1 \exp(\lambda_1 x) + A_2 e_2 \exp(\lambda_2 x) + A_3 e_3 \exp(\lambda_3 x) \tag{14}$$

where the eigenvalues $\lambda_i (i = 1, 2, 3)$ are given by

$$\lambda^3 - (Pe + a_{22})\lambda^2 + (Pe\, a_{22} - a_{11})\lambda + (a_{11}a_{22} - a_{12}a_{21}) = 0 \tag{15}$$

and other coefficients have the forms

$$e_1 = \frac{\lambda_1^2 - Pe\lambda_1 - a_{11}}{a_{12}}, \quad e_2 = \frac{\lambda_2^2 - Pe\lambda_2 - a_{11}}{a_{12}}$$

$$e_3 = \frac{\lambda_3^2 - Pe\lambda_3 - a_{11}}{a_{12}}$$

$$A_1 = \frac{1}{\Delta}\left\{F_1(s)[e_2 w_2 \lambda_3 \exp(\lambda_3) - e_3 w_3 \lambda_2 \exp(\lambda_2)] - F_2(s)\left[\left(1-\frac{\lambda_2}{Pe}\right)\lambda_3 \exp(\lambda_3) - \left(1-\frac{\lambda_3}{Pe}\right)\lambda_2 \exp(\lambda_2)\right]\right\}$$

$$A_2 = \frac{1}{\Delta}\left\{F_1(s)[e_3 w_3 \lambda_1 \exp(\lambda_1) - e_1 w_1 \lambda_2 \exp(\lambda_2)] - F_2(s)\left[\left(1-\frac{\lambda_3}{Pe}\right)\lambda_1 \exp(\lambda_1) - \left(1-\frac{\lambda_1}{Pe}\right)\lambda_3 \exp(\lambda_3)\right]\right\}$$

$$A_3 = \frac{1}{\Delta}\left\{F_1(s)[e_1 w_1 \lambda_2 \exp(\lambda_2) - e_2 w_2 \lambda_1 \exp(\lambda_1)] - F_2(s)\left[\left(1-\frac{\lambda_1}{Pe}\right)\lambda_2 \exp(\lambda_2) - \left(1-\frac{\lambda_2}{Pe}\right)\lambda_1 \exp(\lambda_1)\right]\right\}$$

$$\Delta = \left(1-\frac{\lambda_1}{Pe}\right)[e_2 w_2 \lambda_3 \exp(\lambda_3) - e_3 w_3 \lambda_2 \exp(\lambda_2)] - \left(1-\frac{\lambda_2}{Pe}\right)[e_1 w_1 \lambda_3 \exp(\lambda_3) - e_3 w_3 \lambda_1 \exp(\lambda_1)] - \left(1-\frac{\lambda_3}{Pe}\right)[e_2 w_2 \lambda_1 \exp(\lambda_1) - e_1 w_1 \lambda_2 \exp(\lambda_2)]$$

$w_1 = 1$, $w_2 = 1$, and $w_3 = 1$ for parallel flow, $w_1 = \exp(\lambda_1)$, $w_2 = \exp(\lambda_2)$, and $w_3 = \exp(\lambda_3)$ for counterflow.

The inversion of equations (13) and (14) yields transient temperature distributions of both fluids subject to any inlet temperature variations in the real time-domain. Distinctly, analytical inversion is an overpowering problem. It is rational for one to resort to numerical inversion techniques such as Gaver–Stehfest algorithm [8] and the numerical inversion based on the Fourier series [9], although the numerical inversion is an ill-posed problem. The first algorithm involves only the real variable and the latter the complex variable. By choosing one of these two techniques according to types of inlet temperature variations, one can accurately perform numerical inversion of the Laplace transform [6]. However, the preliminary computation has shown that it takes a large amount of computing time for one to obtain numerical inversion results from equations (13) and (14) if the transformed forms (7) and (9) are utilized, for numerical integration of the modified Bessel functions such as I_0 and K_0 (or I_1 and K_1) need so much time that equations (13) and (14) may be of little use for real-time control and optimal operation of heat exchangers.

An alternative approach to avoid such a difficulty is application of the finite-difference method. It consists of two steps: first equations (3) and (5) are transformed into the Laplace image-domain against the time-variable, and then the transformed ordinary differential equations are replaced by their equivalent difference-equations defined on a number of individual discrete points pertinent to the space-variable. Therefore, the differential equations are reduced to a set of algebraic equations. In this way, one can always obtain the following vectorial matrix forms for $\mathbf{T}_w$ and $\mathbf{T}_s$

$$\mathbf{C}\mathbf{T}_w = \mathbf{b}_w \quad \text{and} \quad \mathbf{D}\mathbf{T}_s = \mathbf{b}_s \tag{16}$$

where vectors $\mathbf{T}_w$ and $\mathbf{T}_s$ are composed of the transformed temperatures T_{wi} ($i = 1, 2, \ldots, M$) and T_{sj} ($j = 1, 2, \ldots, N$), respectively. Matrix $\mathbf{C}$ or $\mathbf{D}$ is determined by finite-difference approximation and numbers of discretized points of a tube wall or the shell, and $\mathbf{b}_w$ or $\mathbf{b}_s$ by the boundary conditions.

On solving equations (16), one finds T_{w1}, T_{w2} and T_{sj} which are needed for determining T_1 and T_2. If grid points M and N are more than 4, it is somewhat difficult to express T_{w1}, T_{w2} and T_{sj} as explicit functions of T_1 and T_2. In these cases, one should also substitute the transformed forms of equations (1) and (2) with the corresponding finite-difference forms. By combining them with equations (16), one can first obtain values of the transformed temperatures in the image-domain by iteration, and then temperature distributions in the real time-domain by the numerical inversion. Such a procedure may be called Hybrid Finite-Difference and Laplace Transform Method. In the simplest case (with enough accuracy) that both the tube wall and the shell are divided into two layers (three grid points) in radial direction, T_{w1} and T_{w2} as well as T_{sj} can be explicitly expressed by the same forms as equations (8) and (10). Here coefficients $\alpha_{1,2}$, $\beta_{1,2}$, and γ are dependent upon finite-difference forms. To ensure the finite-difference approximation the second order of accuracy, the derivatives such as $\mathrm{d}T_w/\mathrm{d}\xi$ and $\mathrm{d}T_s/\mathrm{d}\eta$ are replaced by the central finite-difference and the boundary conditions by three-point finite-difference [10]. Thus, solutions (13) and (14) are still valid even if the finite-difference method is applied to the transformed equations of t_w and t_s. From these transformed solutions, the numerical inversion will accurately and quickly yield the final transient response to any inlet temperature variations.

EFFECT OF RADIAL HEAT CONDUCTION

Solutions (13) and (14) construct the basis of transfer functions for multi-input and multi-output systems. If necessary, the transfer functions can be expressed by a matrix form [11]. In this article the emphasis is put on transient behaviour of heat exchangers in the time-domain to quantitatively estimate the effect of radial heat conduction in a tube wall and in the shell. By means of the above-mentioned numerical inversion techniques, some examples have been calculated.

Figure 2 shows outlet temperature responses t_{1e} and t_{2e} of both fluids to the step temperature change taking place at the entrance of fluid 1 in a counterflow heat exchanger. The curves in the figure clearly illustrate the considerable effect of radial heat conduction in a tube wall on dynamic characteristic. If the Biot number $Bi_1 > 3$, negligence of this type of heat conduction leads to over 50% error of the calculated outlet temperatures. Only if $Bi_1 < 0.05$ can one neglect the radial heat conduction in a wall, i.e. the temperature of the wall is independent of the space variable ξ. Hence, it is recommended that one should first calculate the Biot number, and then decide whether to take the radial heat conduction into account or use the lumped heat capacity method for dynamic analysis of heat exchangers. In addition, the radial heat conduction affects temperature response on the other side. As shown in Fig. 2(b), the delay time of fluid 2 subject to the entrance variation of fluid 1 varies considerably with Bi_1. A bigger value of Bi_1 results in more sluggish outlet response of fluid 2 and such sluggishness is quite plain for the dimensionless time $z < 1$. Without considering the radial heat conduction, transient analysis may conceal this tardy feature.

The curves in Fig. 2 were calculated under extreme values of Bi_{si} and Bi_{so} ($Bi_{si} = 10^8$, $Bi_{so} = 0$) which means no heat loss from the shell to the environment in this example, so that the steady energy balance $R_1(1-t_{1e}) = t_{2e}$ is accurately satisfied. For step inlet variations, the aforementioned two numerical inversion methods were applied and the same results were obtained. If there are oscillatory components in inlet disturbances, as pointed out previously [4], one should prefer the inversion algorithm based on the Fourier series.

U_1 or U_2 is one of the main parameters that manipulate exertion of the effect of radial heat conduction. This effect remarkably increases with decreasing values of U_1 or U_2. For the same Bi_1, the smaller U_1 and U_2, the longer the delay time of temperature response of a fluid corresponding to the inlet change on the other side. As illustrated in Fig. 3, this phenomenon is especially distinct for the span $z < 4$ which covers a strongly transient phase. In this phase, if the radial heat conduction is neglected, the calculation error of t_{2e} becomes over 50% and will be greatly enlarged by reduction of U_1 and U_2. After the dimensionless time $z > 12$, the effect of heat conduction no

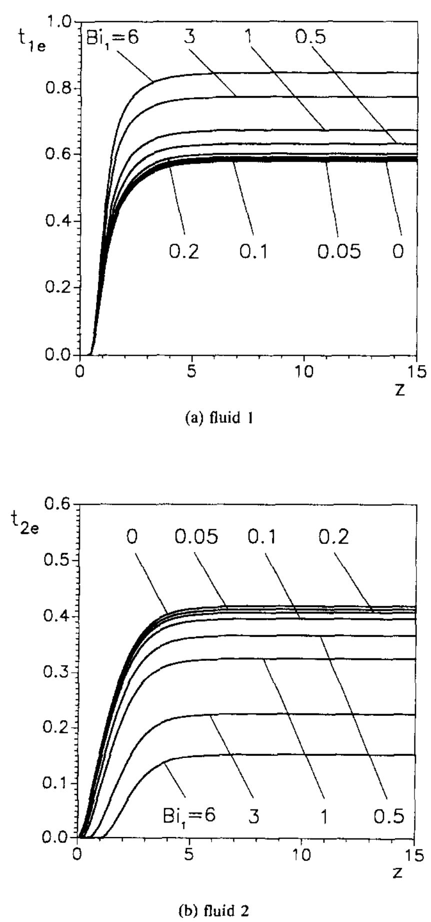

FIG. 2. Transient responses to a step inlet temperature change in a counterflow heat exchanger ($a = 0.9$, $b = 0.9$, $1/\alpha = 0.05$, $1/\beta = 0.02$, $Pe = 20$, $U_1 = U_2 = 1.5$, $U_s = 0.1U_1$, $R_1 = R_\tau = 1$, $R_w = R_s = 0.5$, $Bi_{si} = 10^8$, $Bi_{so} = 0$, $f_1(z) = 1$, $f_2(z) = 0$). (a) Fluid 1, (b) fluid 2.

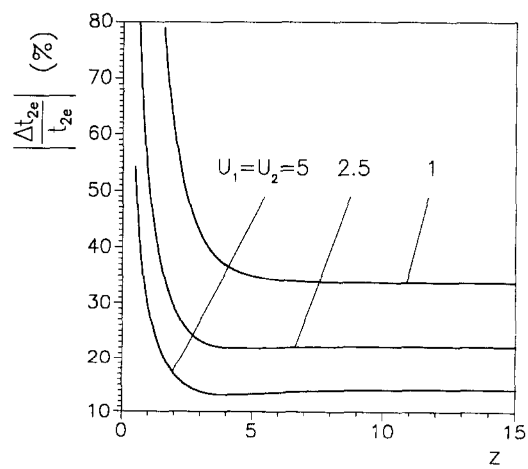

FIG. 3. Effect of the radial heat conduction in a counterflow exchanger ($a = 0.9, b = 0.9, 1/\alpha = 0.05, 1/\beta = 0.02, Pe = 20$, $U_s = 0.1U_1$, $R_1 = R_\tau = 1$, $R_w = 0.6$, $R_s = 0.4$, $Bi_1 = 1$, $Bi_{si} = 10^8$, $Bi_{so} = 0$, $f_1(z) = 1$, $f_2(z) = 0$).

longer varies with the time variable since the heat transfer process has approached the stationary state.

Besides the temperature difference between the outside of the shell and the environment, parameters such as U_s and Bi_{si} as well as Bi_{so} mainly control heat loss from exchangers to the environment. The Biot number Bi_{si} determines whether the temperature difference Δt_s across the solid is larger than that between the inner surface of the shell and the fluid 1 or not, and then the Biot number Bi_{so} prescribes whether Δt_s is larger than the temperature difference between the outside of the shell and the environment. In the example shown in Fig. 4 the remarkable influence of the heat loss arises for $z > 4.0$ and it takes somewhat longer time to reach the steady state. If $Bi_{so} > 0$, the stationary relationship $R_1(1-t_{1e}) = t_{2e}$ is no longer valid and the deviation between $R_1(1-t_{1e})$ and t_{2e} is chiefly controlled by U_s and Bi_{si}.

APPROXIMATION OF RADIAL HEAT CONDUCTION

The lumped heat capacity model has often been used for a tube wall, i.e. temperature profile in a wall has been treated as a function of time only because of its simplicity. Based on this model, radial heat conduction is approximately treated below. Heat conduction resistance of a tube wall can be divided into two parts, either of which is added to the side of fluid 1 or fluid 2, respectively. Therefore, apparent heat transfer coefficients h_1^* and h_2^* are introduced as:

$$\frac{1}{h_1^* A_1} = \frac{1}{h_1 A_1} + \varphi \frac{\delta_w}{\lambda_w A_m}$$

$$\frac{1}{h_2^* A_2} = \frac{1}{h_2 A_2} + (1-\varphi) \frac{\delta_w}{\lambda_w A_m} \tag{17}$$

where coefficient φ lies in the range $0 \leqslant \varphi \leqslant 1$ and the average area A_m is defined as $A_m = (A_1 - A_2)/\ln(A_1/A_2)$ for a circular tube [12]. Thus, apparent shell and tubeside numbers of transfer units are expressed by

$$U_1^* = \frac{U_1}{1+\varphi Bi_1 \dfrac{A_1}{A_m}}, \quad U_2^* = \frac{U_2}{1+(1-\varphi) Bi_2 \dfrac{A_2}{A_m}} \tag{18}$$

By substituting U_1^* and U_2^* for U_1 and U_2, radial heat conduction in the wall can be approximately taken into account with the analytical method [4] based on the lumped heat capacity model for a tube wall, i.e. $t_{w1}(\xi, z) = t_{w2}(\xi, z) = t_w(z)$. As for the coefficient φ, the first intuitive choice may be $\varphi = 0.5$. In fact, calculation has shown that any value of φ around 0.5 leads to no great fluctuation of exit transient responses.

Figure 5 illustrates comparison between accurate and approximate treatment of radial heat conduction and reflects a satisfactory agreement between both approaches, especially within the initial rise-phase and end-phase (approaching a stationary state). Somewhat greater deviation exists in the middle phase of a transient process, which is usually acceptable. By means of the parameters defined in equations (18), hence, one finds a short cut of handling radial heat conduction in a tube wall, and then saves much computing time which is important for real-time control.

CONCLUSIONS

The calculated examples have shown that radial heat conduction exerts a remarkable effect on transient behaviour of heat exchangers and should be taken into account. The effect embodies values of temperature responses and delay time of responses.

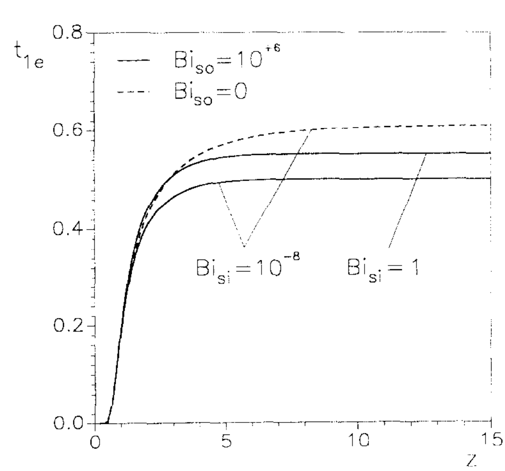

FIG. 4. Influence of heat loss in a counterflow heat exchanger ($a = 0.9$, $b = 0.9$, $1/\alpha = 0.05$, $1/\beta = 0.02$, $Pe = 20$, $U_1 = U_2 = 2$, $U_s = 0.1U_1$, $R_1 = R_\tau = 1$, $R_w = 0.6$, $R_s = 0.4$, $f_1(z) = 1$, $f_2(z) = 0$).

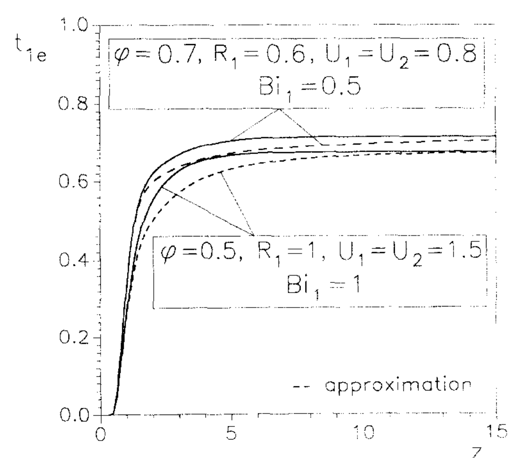

FIG. 5. Comparison between accurate and approximate treatment of radial heat conduction in a counterflow heat exchanger ($a = 0.9$, $b = 0.9$, $1/\alpha = 0.05$, $1/\beta = 0.02$, $Pe = 20$, $U_s = 0.1U_s$, $R_\tau = 1$, $R_w = 0.6$, $R_s = 0.5$, $f_1(z) = 1$, $f_2(z) = 0$).

Only when the Biot number $Bi_\mathrm{I} < 0.05$ can this type of heat conduction be approximately neglected. Heat loss from the shell to the environment depends upon parameters such as U_s, Bi_si, and Bi_so. It can also change the dynamic characteristic of exchangers.

Using the apparent shell and tubeside number of transfer units defined in equation (18), one can approximately treat the effect of radial heat conduction in the wall. Such treatment has taken advantage of the conventional lumped model for a tube wall and reduced much of computing time.

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